

Problem 1 (answer on page 1 of the booklet)

Find the domain and the range of the function $f(x, y, z) = \frac{1}{\ln \sqrt{4-x^2-y^2-z^2}}$. Determine if the domain of f is an open region, a closed region or neither? Also, determine if the domain is bounded or unbounded. Also, describe the level curves of f . (10 pts)

Problem 2 (answer on page 2 of the booklet)

Find the equations of the tangent plane and normal line to the curve of intersection of the paraboloid $z = x^2 + y^2$ and the ellipsoid $x^2 + 4y^2 + z^2 = 9$ at the point $(1, -1, 2)$. (16 pts)

Problem 3 (answer on page 3 of the booklet)

Find all local maxima, local minima and saddle points for $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$. (16 pts)

Problem 4 (answer on page 4 of the booklet)

For each of the following limits, say if it exists or no, justifying your answer. (7+8+8 pts)

a) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3}{x^2+y^2} \sin\left(\frac{1}{y}\right)$

b) $\lim_{(x,y) \rightarrow (1,-1)} \frac{x^2-y^2}{1+xy}$

c) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3y}{x^6+y^2}$

Problem 5 (answer on pages 5 and 6 of the booklet)

Let $x = \ln(r + s)$, $y = \cos^{-1}\left(\frac{r}{s}\right)$, $z = \sqrt{s - r}$ and $w = \tan\left(\frac{x}{y}\right) e^{yz}$

- (i) Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ at $(r, s) = (0, 1)$. (7 pts)
- (ii) Find the directions of zero change in w at the point $(r, s) = (0, 1)$ (6 pts)
- (iii) Find a line normal to the surface $w(r, s) = \tan\left(\frac{\ln 2}{2\pi}\right)$ in the $rs - plane$. (8 pts) (Hint: you may need the fact that $\cos^{-1}(1) = 2\pi$)

Problem 6 (answer on the last page of the booklet and its back)

The two parts of the following question are independent.

- (i) Let $w = x + y$ where $x = \ln(\sec^2 \frac{t}{2})$ and $y = \sin t$. Find α such that $\left. \frac{dw}{dt} \right|_{t=\alpha} = 1$. (7 pts)
- (ii) By how much will $f(x, y, z) = \ln \sqrt{x^2 + y^2 + z^2}$ change if the point $P(x, y, z)$ moves from $P_0(3, 4, 12)$ a distance $ds = 0.1$ unit in the direction of $3\vec{i} + 6\vec{j} - 2\vec{k}$? (7 pts)

Good Luck!

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